كلية: الحاسبات و الذكاء الصناعي

المستوى: الثاني

الفصل الدراسي: الأول ٢٠٢٢/٢٠٢١

تاريخ الأختبار: ٢٠٢٢/١/١٧

نموذج اجابة : ورقة كاملة

اسم المقرر: Linear Algebra الجبر خطي

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صورة من الأسئلة

Benha University
1^{st} Term (January 2022) Final Exam
Course Code: Level: 2^{nd} level
Subject: Linear Algebra

Faculty of Computers & Artificial Intelligence
Date: 17 / 01 /2022
Time: 2 hrs.
Total Marks: 50 Marks
Examiner(s): Pof.Dr. Reda Gamal
Dr/ Mostafa Hassan

Answer the following questions:

Q1 (13Marks)

a) Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$ by reducing it to a row – echelon form.

b) Is the set $W = \{(a, 0, c): a, c \in \mathbb{R}\}$ a subspace of $\mathbb{R}^3$ with the standard operations? Explain your answer.

Q2 (13Marks)

a) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 - x_3)$

(1) Show that T is a Linear transformation

(2) Find Ker (T).

b) Is the polynomial $2t^2$ belongs to span $\{t^3 - t + 1, 3t^3 + 2t, t^3\}$. Explain your answer.

Q3 (13Marks)

a) Show that the set $S = \{(1, 1), (1, -1)\}$ is a basis for $\mathbb{R}^2$ with the standard operations.

b) Show that the set $S = \{x_1, x_2, x_3, x_4\}$ in a vector space $\mathbb{R}^3$ where

$x_1 = (1, 2, -1) \quad x_2 = (1, -2, 1) \quad x_3 = (-3, 2, -1) \quad x_4 = (2, 0, 0)$

are linearly dependent.

Q4 (11Marks)

Find the eigenvalues and corresponding eigenvectors of the following matrix

$$A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}.$$

الأجـــابــة

Faculty of Computers and A.I.

Linear Algebra 2nd Level : 17/01/2022

Model Answer

Q1 a)

$$\begin{pmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{pmatrix} \xrightarrow[-2R_1+R_3]{R_1+R_2} \begin{pmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 3 & 5 \end{pmatrix} \xrightarrow{-R_2+R_3} \begin{pmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 4 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{2}R_3} \begin{pmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \quad \therefore Rk \begin{pmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{pmatrix} = 3$$

b) yes: because

(1) Let $W_1 = (a_1, 0, c_1) \in W$, $W_2 = (a_2, 0, c_2) \in W$ $\Longrightarrow W_1 + W_2 = (a_1 + a_2, 0, c_1 + c_2) \in W$

(2) $\lambda W_1 = (\lambda a_1, 0, \lambda c_1) \in W$, λ - Scalar

Q2) a) (1) Let $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$

$\Rightarrow x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$\lambda x = (\lambda x_1, \lambda x_2, \lambda x_3)$

$\Rightarrow$

1) $T(x+y) = (x_1 + y_1 + x_2 + y_2, x_2 + y_2 - x_3 - y_3)$

$= (x_1 + x_2 + y_1 + y_2, x_2 - x_3 + y_2 - y_3)$

$= (x_1 + x_2, x_2 - x_3) + (y_1 + y_2, y_2 - y_3)$

$= T(x) + T(y)$.

2) $T(\lambda x) = (\lambda x_1 + \lambda x_2, \lambda x_2 - \lambda x_3)$

$= (\lambda(x_1 + x_2), \lambda(x_2 - x_3))$

$= \lambda(x_1 + x_2, x_2 - x_3) = \lambda T(x)$.

from 1) and 2) T is Linear Transformation.

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(a) (2) $\ker(T)=\{(x_1,x_2,x_3) : T(x_1,x_2,x_3)=(0,0$

$\Rightarrow (x_1+x_2, x_2-x_3)=(0,0)$

$x_1+x_2=0$ $\quad$ 1 1 0

$x_2-x_3=0$ $\quad$ 0 1 -1 $\quad$ $Rk\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & -1\end{pmatrix}=2$

Let $x_3=\alpha \Rightarrow x_2=\alpha \Rightarrow x_1=-\alpha$

$\therefore \ker(T)=\{\alpha(1,1,-1), \alpha\in\mathbb{R}\}$.

Q2) b) Let

$2t^2=c_1(t^3-t+1)+c_2(3t^3+2t)+c_3t^3$

$=(c_1+3c_2+c_3)t^3+0t^2+(-c_1+2c_2)t+c_1$

$\Rightarrow 2=0$ Contradiction

$\Rightarrow 2t^2\notin \text{Span}\{t^3-t+1, 3t^3+2t, t^3\}$.

Q3). a) 1) $(1,1),(1,-1)$ Linearly independent because

$c_1(1,1)+c_2(1,-1)=(0,0)$

$\Rightarrow c_1+c_2=0$, $c_1-c_2=0$ $\Rightarrow c_1=c_2=0$.

2) $\forall (x,y)\in\mathbb{R}^2$ $(x,y)=c_1(1,1)+c_2(1,-1)$

$\Rightarrow c_1+c_2=x$, $c_1-c_2=y$ $\Rightarrow 2c_1=x+y \Rightarrow c_1=\frac{1}{2}(x+y)$

$c_2=\frac{1}{2}(x-y)$

$\Rightarrow \mathbb{R}^2=\text{Span}\{(1,1),(1,-1)\}$

$\Rightarrow$ S is a basis for $\mathbb{R}^2$.

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Q3) b)

$$c_1(1,2,-1)+c_2(1,-2,1)+c_3(-3,2,-1)+c_4(2,0,0)=(0,0,0)$$

$\Rightarrow$

$$(c_1+c_2-3c_3+2c_4,\ 2c_1-2c_2+2c_3,\ -c_1+c_2-c_3)=(0,0,0)$$

$$\therefore \begin{aligned} c_1+c_2-3c_3+2c_4&=0\\ 2c_1-2c_2+2c_3&=0\\ -c_1+c_2-c_3&=0 \end{aligned} \quad \begin{pmatrix} 1 & 1 & -3 & 2\\ 2 & -2 & 2 & 0\\ -1 & 1 & -1 & 0 \end{pmatrix} \xrightarrow[R_1+R_3]{-2R_1+R_2}$$

$$\begin{pmatrix} 1 & 1 & -3 & 2\\ 0 & -4 & 8 & -4\\ 0 & 2 & -4 & 2 \end{pmatrix} \xrightarrow[\frac{1}{2}R_3]{-\frac{1}{4}R_2} \begin{pmatrix} 1 & 1 & -3 & 2\\ 0 & 1 & -2 & 1\\ 0 & 1 & -2 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2-R_3} \begin{pmatrix} 1 & 1 & -3 & 2\\ 0 & 1 & -2 & 1\\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{aligned} c_1+c_2-3c_3+2c_4&=0\\ c_2-2c_3+c_4&=0 \end{aligned}$$

$R(K)=2$

put $\Rightarrow c_3=\alpha,\ c_4=\beta$

$c_2=2\alpha-\beta,\quad c_1=\alpha-\beta.$

say if $\alpha=1,\ \beta=0$

$\Rightarrow c_1=1,\ c_2=2,\ c_3=1,\ c_4=0$

Q4) The ch. eq is

$$|C-\lambda I|=0 \quad \begin{vmatrix} -3-\lambda & -7 & -5\\ 2 & 4-\lambda & 3\\ 1 & 2 & 2-\lambda \end{vmatrix}=0$$

$$\Rightarrow \lambda^3-3\lambda^2+3\lambda+1=0 \Rightarrow \lambda_1=\lambda_2=\lambda_3=1$$

at $\lambda=1$, we get

$$(C-I)X=0 \Rightarrow \begin{bmatrix} -4 & -7 & -5\\ 2 & 3 & 3\\ 1 & 2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow Rk(C-I)=2$, put $x_3=\alpha$, $x_1+2x_2+x_3=0$, $x_2-x_3=0$

$\Rightarrow x_1=-3\alpha,\ x_2=\alpha,\ x_3=\alpha \ \therefore X=[x_1,x_2,x_3]^T$

$=\alpha[-3,1,1]^T$

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