كلية الحاسبات والمعلومات

المستوي الاول

الفصل الدراسي الاول

2022-2021

تاريخ الامتحان: 2022/1/30

نموذج اجابة ورقة كاملة

المادة: تأهيلى الرياضيات

أستاذ المادة : أ.د / أحمد مصطفي عبدالباقي مجاهد

استاذ بقسم الرياضيات بكلية العلوم ببنها

صورة من الاسئلة

Benha University
1st Term (January 2022) Final Exam
Level: 1st level
Subject: Qualifying Mathematics

Faculty of Computers & AI
Date: 30 /1 /2022
Time: 2 hrs.
Total Marks: 50 Marks
Examiner(s): Dr. Ahmed Megahed

Answer the following questions:

Q1) Find the modulus and the principle amplitude of each of the following complex

$a)\ Z_1 = -4i$ $\qquad\qquad$ $b)\ Z_2 = 6$

Q2) Express each of the following complex numbers in trigonometric form :

$a)\ Z_1 = -4 - 4\sqrt{3}\,i$ $\qquad\qquad$ $b)\ Z_2 = 2\sqrt{3} - 2i$

Q3) Without expanding the determinant, prove that :

i) $\begin{vmatrix} 3 & 5 & 7 \\ 2 & -1 & 6 \\ 4 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 5 & 7 \\ -1 & 4 & -4 \\ 4 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 5 & 7 \\ 1 & 3 & 2 \\ -2 & -3 & 2 \end{vmatrix} = \text{zero}$

ii) $\begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 39 & 9 & 70 \end{vmatrix} = 1$

Q4) Find the value of the constant k which makes $(x-2)$ one of the roots of the determinant $\begin{vmatrix} x+1 & 1 & -3 \\ 2 & 5 & x-1 \\ 1 & -4 & x+k \end{vmatrix}$

Q5) Find the multiplicative inverse of the matrix $\begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix}$

Q6) If $A = \begin{pmatrix} 6 & 2 \\ -8 & -3 \end{pmatrix}$, then prove that $(A^t)^{-1} = (A^{-1})^t$.

GOOD LUCK,

Dr. Ahmed Megahed

Model Answer

Q1)

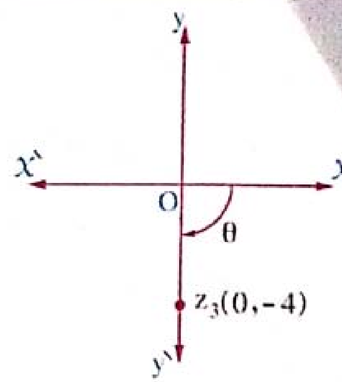
$\because z_3 = -4i$ $\quad\therefore x = 0 \;,\; y = -4$

$\therefore |z_3| = \sqrt{0+16} = 4$ length unit

$, \because x = 0 \;,\; y < 0$ $\quad\therefore \theta = \frac{-\pi}{2}$

$\therefore$ The modulus of the number $z_3 = 4$

, the principle amplitude of the number $z_3 = \frac{-\pi}{2}$



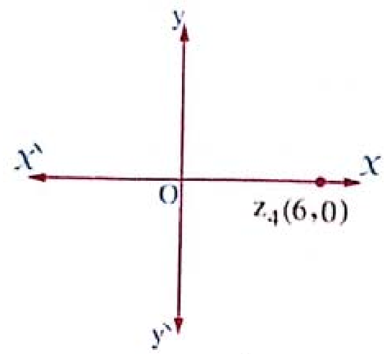
$\because z_4 = 6$ $\quad\therefore x = 6 \;,\; y = 0$

$\therefore |z_4| = \sqrt{36+0} = 6$ length unit

$, \because x > 0 \;,\; y = 0$ $\quad\therefore \theta = 0$

$\therefore$ The modulus of the number $z_4 = 6$ length unit

and the principle amplitude of the number $z_4 = 0$



Q2)

a)

$z_2 = -4 - 4\sqrt{3}\,i$ $\quad\therefore x = -4 \;,\; y = -4\sqrt{3}$

$\therefore |z_2| = r = \sqrt{x^2 + y^2} = \sqrt{16+48} = 8$

$\because x < 0 \;,\; y < 0$ $\quad\therefore \theta$ lies in the 3^{rd} quadrant

$\therefore \theta = -\pi + \tan^{-1}\left(\frac{y}{x}\right) = -\pi + \tan^{-1}\left(\frac{-4\sqrt{3}}{-4}\right)$

$= -\pi + \tan^{-1}\left(\sqrt{3}\right) = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$

$, \because z_2 = r(\cos\theta + i\sin\theta)$

$\therefore z_2 = 8\left(\cos\left(\frac{-2\pi}{3}\right) + i\sin\left(\frac{-2\pi}{3}\right)\right)$

b)

$\therefore x = 2\sqrt{3}\ ,\ y = -2$

$z_3 = 2\sqrt{3} - 2i$

$\therefore |z_3| = r = \sqrt{x^2 + y^2} = \sqrt{12 + 4} = 4$

$\therefore \theta$ lies in the 4^{th} quadrant

$\because x > 0\ ,\ y < 0$

$\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2}{2\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = \frac{-\pi}{6}$

$\therefore z_3 = r(\cos\theta + i\sin\theta) = 4\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)$

Q3)

i)

$$\text{L.H.S.} = \begin{vmatrix} 3 & 5 & 7 \\ 2 & -1 & 6 \\ 4 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 5 & 7 \\ -1 & 4 & -4 \\ 4 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 5 & 7 \\ 1 & 3 & 2 \\ -2 & -3 & 2 \end{vmatrix}$$ (Adding the first and second determinants where $R_1 = R_3$)

$$= \begin{vmatrix} 3 & 5 & 7 \\ 1 & 3 & 2 \\ 4 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 5 & 7 \\ 1 & 3 & 2 \\ -2 & -3 & 2 \end{vmatrix}$$ (Adding the two determinants where $C_2 = C_3$)

$$= \begin{vmatrix} 7 & 5 & 7 \\ 2 & 3 & 2 \\ 2 & -3 & 2 \end{vmatrix}$$ (Note that : $C_1 = C_3$)

$\therefore$ The determinant = zero = R.H.S.

ii)

$$= \begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 39 & 9 & 70 \end{vmatrix} \left(R_1 \times (-3) + R_3\right) = \begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 0 & 0 & 1 \end{vmatrix} \left(C_2 \times (-4) + C_1\right)$$

$$= \begin{vmatrix} 1 & 3 & 23 \\ 2 & 7 & 53 \\ 0 & 0 & 1 \end{vmatrix} \left(R_1 \times (-2) + R_2\right)$$

$$= \begin{vmatrix} 1 & 3 & 23 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{vmatrix}$$ (The determinant in the triangular from) $= 1 \times 1 \times 1 = 1 =$ R.H

Q4)

If $(X-2)$ is a factor of the determinant, then $X = 2$ makes the value of the determinant = zero, by substituting by $X = 2$, then the required is finding the value of k which makes the resulting determinant vanishes.

i.e. $\begin{vmatrix} 3 & 1 & -3 \\ 2 & 5 & 1 \\ 1 & -4 & 2+k \end{vmatrix} = 0$ «By adding C_1 and C_3»

$\therefore \begin{vmatrix} 3 & 1 & 0 \\ 2 & 5 & 3 \\ 1 & -4 & 3+k \end{vmatrix} =$ zero $\quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

By expanding the determinant using the third column

$\therefore -3\,(-12-1) + (3+k)\,(15-2) = 0$

$\therefore -3 \times -13 + (k+3) \times 13 = 0$ (Divide both sides by 13)

$\therefore 3 + k + 3 = 0 \qquad \therefore k = -6$

Q5)

$$A = x\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \therefore A = xI$$

$$\therefore A^{-1} = (XI)^{-1} = \frac{1}{X} I$$

$$\therefore A^{-1} = \frac{1}{x}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Q6)

$$A^t = \begin{pmatrix} 6 & -8 \\ 2 & -3 \end{pmatrix} \qquad \therefore (A^t)^{-1} = \frac{1}{-2}\begin{pmatrix} -3 & 8 \\ -2 & 6 \end{pmatrix}$$

$$(A^t)^{-1} = \begin{pmatrix} \frac{3}{2} & -4 \\ 1 & -3 \end{pmatrix} \qquad (1)$$

$$A^{-1} = \begin{pmatrix} \frac{3}{2} & 1 \\ -4 & -3 \end{pmatrix} \qquad \therefore (A^{-1})^t = \begin{pmatrix} \frac{3}{2} & -4 \\ 1 & -3 \end{pmatrix} \qquad (2)$$

From (1) , (2) : $\therefore (A^t)^{-1} = (A^{-1})^t$

Dr. Ahmed Mostafa Megahed