Discrete Mathematics and Its Applications

Section 1

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Ch1*: *The Foundations: Logic and Proofs

Section 1.1

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

- **a)** Boston is the capital of Massachusetts.
- **b)** Miami is the capital of Florida.
- **c)** 2 + 3 = 5.
- **d)** 5 + 7 = 10.
- **e)** $x + 2 = 11$.
- **f)** Answer this question.

solution

1. Propositions must have clearly defined truth values, so a proposition must be a declarative sentence with no free variables.

- **a)** This is a true proposition.
- **b)** This is a false proposition (Tallahassee is the capital).
- c) This is a true proposition.
- **d)** This is a false proposition.
- e) This is not a proposition (it contains a variable; the truth value depends on the value assigned to x).
- **f)** This is not a proposition, since it does not assert anything.

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$
d) $p \rightarrow \neg q$ *(ass)* **e)** $\neg q \rightarrow p$ *(ass)* **f)** $\neg p \rightarrow \neg q$ *(ass)*
g) $p \leftrightarrow \neg q$ **h)** $\neg p \wedge (p \vee \neg q)$

solution

9. This is pretty straightforward, using the normal words for the logical operators.

a) Sharks have not been spotted near the shore.

b) Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.

c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.

g) Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.

h) Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore. Note that we were able to incorporate the parentheses by using the word "either" in the second half of the sentence.

11. Let *p* and *q* be the propositions

p : It is below freezing.

q : It is snowing.

Write these propositions using *p* and *q* and logical connectives (including negations).

- **a)** It is below freezing and snowing.
- **b)** It is below freezing but not snowing.
- **c)** It is not below freezing and it is not snowing.
- **d)** It is either snowing or below freezing (or both).
- **e)** If it is below freezing, it is also snowing.
- **f)** Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
- **g)** That it is below freezing is necessary and sufficient for it to be snowing.

solution

11\. a) Here we have the conjunction $p \wedge q$.

b) Here we have a conjunction of p with the negation of q, namely $p \wedge \neg q$. Note that "but" logically means the same thing as "and."

c) Again this is a conjunction: $\neg p \wedge \neg q$.

d) Here we have a disjunction, $p \vee q$. Note that $\vee$ is the inclusive *or*, so the "(or both)" part of the English sentence is automatically included.

e) This sentence is a conditional statement, $p \rightarrow q$.

f) This is a conjunction of propositions, both of which are compound: $(p \vee q) \wedge (p \rightarrow \neg q)$.

g) This is the biconditional $p \leftrightarrow q$.

17. Determine whether each of these conditional statements is true or false.

a) If 1 + 1 = 2, then 2 + 2 = 5.

b) If 1 + 1 = 3, then 2 + 2 = 4.

c) If 1 + 1 = 3, then 2 + 2 = 5.

d) If monkeys can fly, then 1 + 1 = 3.

Solution

17. In each case, we simply need to determine the truth value of the hypothesis and the conclusion, and then use the definition of the truth value of the conditional statement. The conditional statement is true in every case except when the hypothesis (the "if" part) is true and the conclusion (the ''then" part) is false.

a) Since the hypothesis is true and the conclusion is false, this conditional statement is false.

b) Since the hypothesis is false and the conclusion is true, this conditional statement is true.

c) Since the hypothesis is false and the conclusion is false, this conditional statement is true. Note that the conditional statement is false in both part **(b)** and part (**c);** as long as the hypothesis is false, we need look no further to conclude that the conditional statement is true.

d) Since the hypothesis is false, this conditional statement is true

33. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \to (p \oplus q)$

b) $(p \oplus q) \to (p \wedge q)$ (ass)

c) $(p \vee q) \oplus (p \wedge q)$ (ass)

d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$ (ass)

f) $(p \oplus q) \to (p \oplus \neg q)$ (ass)

Solution 33

a)*$(p \vee q) \rightarrow (p \oplus q)$*

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

d) *$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$*

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

43. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001

b) 1111 0000, 1010 1010

c) 00 0111 0001, 10 0100 1000 *(ass)*

d) 11 1111 1111, 00 0000 0000 *(ass)*

Solution

43.

a) bitwise *OR* = 111 1111; bitwise *AND*= 000 0000; bitwise *XOR* = 1111111

b) bitwise *OR* = 1111 1010; bitwise *AND*= 1010 0000; bitwise *XOR* = 0101 1010

Thank you